UNIVERSITY OF TARTU

ESTIMATION UNDER RESTRICTIONS BUILT UPON BIASED INITIAL ESTIMATORS

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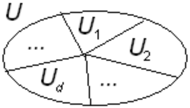
Let the finite population U be divided into D non-overlapping domains U_d,
$d \in \mathcal{D} = \{1, 2, ..., D\}$.



- We are interested in some domain parameters, for example domain totals, $t^d = \sum_{i \in U_d} y_i$ with y_i being the value of study variable for object i.
- It is natural that domain totals sum up to the population total, $\sum_{d=1}^{D} t^d = t = \sum_{i \in U} y_i$.

Relationships between population parameters do not necessarily hold for the estimates in a sample!

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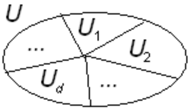

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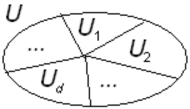

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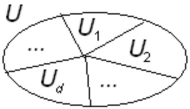

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Cases of inconsistency

- Aggregated from registers population statistics and obtained from the sampling survey domain estimates. *Domain estimates from the survey do not sum up to the totals available from the registers.*
- The multi-survey situation: some study variables are common in two or more surveys. *Domain estimates from one survey do not sum up to the estimates of larger domains (or population totals) from another survey.*
- Domains themselves and the population total may be estimated by conceptually different estimators in the same survey. *As a result, the domain totals do not sum up to the population total, or to the relevant larger domains.*

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Ideas

- It is known that any auxiliary information incorporated into estimators may increase precision of these estimators
- Known relationships between population parameters is a kind of the auxiliary information.
- If one could use this information in the estimators, one were able to make estimators more accurate and force them to satisfy desired restrictions.

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This work is based on earlier results,

- the general restriction (GR) estimator (*Knottnerus 2003*),
- the GR estimator elaborated for domains (*Sõstra 2007, Sõstra ja Traat 2009*).

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- All estimators and their properties are elaborated in general and can be applied for any sampling design.
- Since domain estimation is a multivariate problem, then the matrix technique is used; there are vectors of estimators under consider. The accuracy of these vectors are measured with the MSE matrices.
- The design based approach is used.
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Notations

Let $\boldsymbol{\theta} = (\theta_1, ..., \theta_k)'$ be the parameter vector under study that satisfies linear restrictions:

$$\mathbf{R}\boldsymbol{\theta} = \mathbf{0}, \qquad (1)$$

where $\mathbf{R}$ is an $r \times k$ matrix of rank r.

- For the domain's case,

  $\mathbf{R} = (1, 1, ..., 1, -1) : 1 \times (D + 1)$ and $\boldsymbol{\theta} = (t_y^1, t_y^2, ..., t_y^D, t_y)'$,

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GR for the unbiased $\hat{\boldsymbol{\theta}}$

The GR estimator for the unbiased initial estimators $\hat{\theta} = (\hat{\theta}_1, ..., \hat{\theta}_k)'$ (Knottnerus, 2003) that satisfies (1) has the form

$$\hat{\theta}_{GR} = (\mathbb{I} - \mathbf{KR})\hat{\theta} \qquad (2)$$

with

$$\mathbb{C}\mathrm{ov}(\hat{\theta}_{GR}) = (\mathbb{I} - \mathbf{KR})\mathbf{V}, \qquad (3)$$

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$$\mathbf{K} = \mathbf{VR}'(\mathbf{RVR}')^{-1} \qquad (4)$$

and $\mathbf{V} = \mathbb{C}\mathrm{ov}(\hat{\theta})$ such that $\mathbf{RVR}'$ can be inverted.

The estimator $\hat{\theta}_{GR}$ is **unbiased** and has the **minimum variance** property.

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- Knottnerus' GR estimator is **not optimal** with **biased** initial estimators.
- Three new GR estimators **handling bias** $b = \mathbb{E}(\hat{\theta}) - \theta$ are proposed.
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Estimator GR1

Proposition 2.2

The estimator

$$\hat{\boldsymbol{\theta}}_{GR1} = (\mathbb{I} - \mathbf{KR})(\hat{\boldsymbol{\theta}} - \boldsymbol{b}), \qquad (5)$$

with $\mathbf{K} = \mathbf{VR}'(\mathbf{RVR}')^{-1}$ is unbiased for $\boldsymbol{\theta}$. Its variance is

$$\mathbb{C}\text{ov}(\hat{\boldsymbol{\theta}}_{GR1}) = (\mathbb{I} - \mathbf{KR})\mathbf{V}, \qquad (6)$$

and it is the optimal estimator among all linear estimators in $(\hat{\boldsymbol{\theta}} - \boldsymbol{b})$ that satisfy restriction equation (1).

Estimator GR2

Proposition 2.3

The estimator, satisfying restrictions (1), is

$$\hat{\theta}_{GR2} = (\mathbb{I} - \mathbf{K}^*\mathbf{R})\hat{\theta}, \tag{7}$$

where $\mathbf{K}^* = \mathbf{M}\mathbf{R}'(\mathbf{R}\mathbf{M}\mathbf{R}')^{-1}$ and $\mathbf{M} = \mathbb{MSE}(\hat{\theta})$. The bias of the $\hat{\theta}_{GR2}$ is

$$\boldsymbol{b}(\hat{\theta}_{GR2}) = (\mathbb{I} - \mathbf{K}^*\mathbf{R})\boldsymbol{b}, \tag{8}$$

and the mean square error matrix is

$$\mathbb{MSE}(\hat{\theta}_{GR2}) = (\mathbb{I} - \mathbf{K}^*\mathbf{R})\mathbf{M}. \tag{9}$$

Furthermore, $$\mathbb{MSE}(\hat{\theta}_{GR2}) \leq \mathbf{M} \tag{10}$$
in the sense of Löwner ordering.

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$$\hat{\boldsymbol{\theta}}_{GR3} = (\mathbb{I} - \mathbf{K}^*\mathbf{R})(\hat{\boldsymbol{\theta}} - \boldsymbol{b}) \qquad (11)$$

with $\mathbf{K}^* = \mathbf{M}\mathbf{R}'(\mathbf{R}\mathbf{M}\mathbf{R}')^{-1}$ satisfies restrictions (1) and is unbiased for $\boldsymbol{\theta}$. It's MSE is

$$\mathbb{MSE}(\hat{\boldsymbol{\theta}}_{GR3}) = (\mathbb{I} - \mathbf{K}^*\mathbf{R})\mathbf{V}(\mathbb{I} - \mathbf{K}^*\mathbf{R})'. \qquad (12)$$

Furthermore,

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$$\hat{\boldsymbol{\theta}}_{GR1} = (\mathbb{I} - \mathsf{KR})(\hat{\boldsymbol{\theta}} - \boldsymbol{b}),$$

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Proposition 2.4

The mean square error matrices of the restriction estimators $\hat{\boldsymbol{\theta}}_{GR1}$, $\hat{\boldsymbol{\theta}}_{GR2}$, $\hat{\boldsymbol{\theta}}_{GR3}$ and the initial estimator $\hat{\boldsymbol{\theta}}$ can be ordered (in the sense of Löwner ordering) as following:

$$\mathbb{MSE}(\hat{\boldsymbol{\theta}}_{GR1}) \leq \mathbb{MSE}(\hat{\boldsymbol{\theta}}_{GR3}) \leq \mathbb{MSE}(\hat{\boldsymbol{\theta}}_{GR2}) \leq \mathbb{MSE}(\hat{\boldsymbol{\theta}}). \qquad (14)$$

Remarks to GR1-GR3

- GR estimators are general:
  - applicable for any initial estimator-vector $\hat{\theta}$,
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Simulation study

Population:	Data of the healthcare personnel of Estonia, $N = 21764$
Study variables:	Hourly wage (continuous) Physician (binary)
Auxiliary information:	ID of healthcare institution Age Sex Education level (1..5) Domain indicator d ($d = 1, 2, 3, 4$)

Data description

The population is divided into 4 domains by the type of the healthcare institution:

Table: Population and domain sizes

Domain	no. of laborers	%
1	10863	49.9
2	6742	31.0
3	3139	14.4
4	1020	4.7
Population	21764	100

Data description

- Two sample designs were applied for the population frame, SI and MN.
- For both designs 5000 independent samples were drawn with $n = 400$.
- There is no any empty domain sample through simulations.

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The restriction matrix $\mathbf{R}$ and the restriction equation for the vector of true totals $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_P)'$, are:

$$\mathbf{R} = (1, 1, 1, 1, -1), \mathbf{R}\boldsymbol{\theta} = 0.$$

The initial estimator vector:

$$\hat{\boldsymbol{\theta}} = (\hat{t}^1_{greg\text{-}P}, \hat{t}^2_{greg\text{-}D}, \hat{t}^3_{syn\text{-}P}, \hat{t}^4_{syn\text{-}P}, \hat{t}_{greg})'.$$

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Consistency of GR, SI

Estimator	Sample	Domain d				Population	$\mathbf{R}\hat{\theta} =$
		$d = 1$	$d = 2$	$d = 3$	$d = 4$		
Initial	1	2450.3	1428.3	884.1	178.3	4706.2	234.9
	2	2291.8	1878.7	1003.3	191.0	5160.7	204.1
	3	2777.1	1460.7	917.8	192.5	5019.6	328.6
GR1	1	2523.2	1459.8	544.0	151.2	4678.3	0.0
	2	2382.3	1917.8	662.2	163.7	5126.0	0.0
	3	2796.2	1469.0	580.7	166.4	5012.3	0.0
GR2	1	2425.6	1417.6	707.4	165.0	4715.7	0.0
	2	2270.2	1869.4	849.8	179.5	5168.9	0.0
	3	2742.5	1445.8	670.7	173.9	5032.9	0.0
GR3	1	2463.7	1434.1	643.6	159.6	4701.0	0.0
	2	2308.4	1885.9	785.9	174.1	5154.3	0.0
	3	2780.6	1462.3	606.8	168.6	5018.2	0.0

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	2	2291.8	1878.7	1003.3	191.0	5160.7	204.1
	3	2777.1	1460.7	917.8	192.5	5019.6	328.6
GR1	1	2523.2	1459.8	544.0	151.2	4678.3	0.0
	2	2382.3	1917.8	662.2	163.7	5126.0	0.0
	3	2796.2	1469.0	580.7	166.4	5012.3	0.0
GR2	1	2425.6	1417.6	707.4	165.0	4715.7	0.0
	2	2270.2	1869.4	849.8	179.5	5168.9	0.0
	3	2742.5	1445.8	670.7	173.9	5032.9	0.0
GR3	1	2463.7	1434.1	643.6	159.6	4701.0	0.0
	2	2308.4	1885.9	785.9	174.1	5154.3	0.0
	3	2780.6	1462.3	606.8	168.6	5018.2	0.0

Consistency of GR, MN

Estimator	Sample	Domain d				Population	$\mathbf{R}\hat{\theta} =$
		$d = 1$	$d = 2$	$d = 3$	$d = 4$		
Initial	1	2666.0	1676.8	952.5	178.1	4883.7	589.7
	2	2419.3	1423.5	931.1	169.9	4725.0	218.8
	3	2674.4	1241.9	908.7	164.7	4604.8	384.8
GR1	1	2533.9	1613.3	622.7	154.0	4923.8	0.0
	2	2502.3	1463.4	591.2	142.9	4699.8	0.0
	3	2661.1	1235.5	573.3	139.0	4608.8	0.0
GR2	1	2600.0	1645.1	513.6	145.0	4903.7	0.0
	2	2394.8	1411.7	768.3	157.6	4732.4	0.0
	3	2631.3	1221.1	622.4	143.1	4617.9	0.0
GR3	1	2640.5	1664.6	446.9	139.5	4891.4	0.0
	2	2435.3	1431.2	701.5	152.1	4720.1	0.0
	3	2671.8	1240.6	555.6	137.5	4605.6	0.0

Consistency of GR, MN

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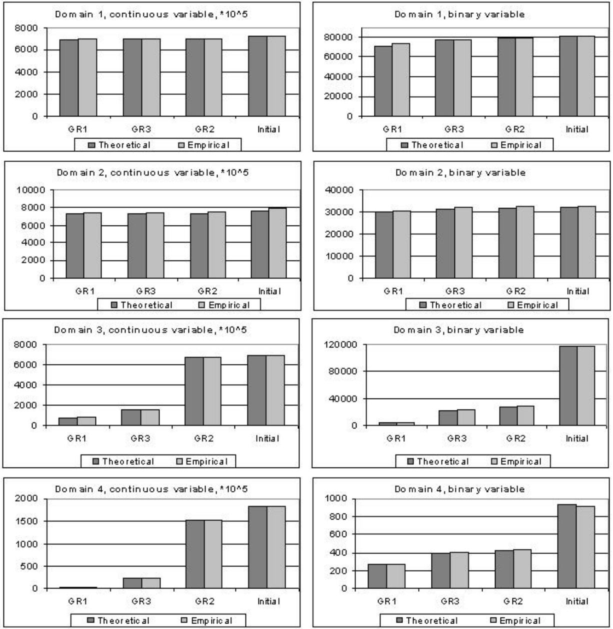
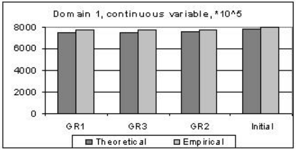
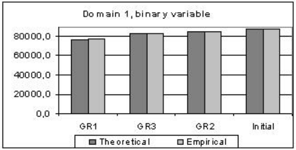
MSEs, SI case

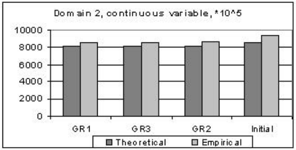


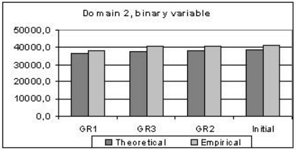
Domain 1, continuous variable, *10^5
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 1, binary variable
80000
60000
40000
20000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 2, continuous variable, *10^5
10000
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 2, binary variable
40000
30000
20000
10000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 3, continuous variable, *10^5
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 3, binary variable
120000
80000
40000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 4, continuous variable, *10^5
2000
1500
1000
500
0
GR1
GR3
GR2
Initial
Theoretical
Empirical
Domain 4, binary variable
1000
800
600
400
200
0
GR1
GR3
GR2
Initial
Theoretical
Empirical

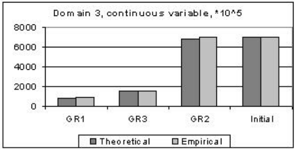
MSEs, MN case

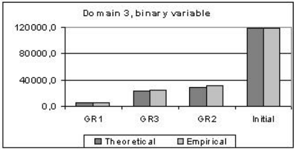

Domain 1, continuous variable, *10^5
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical

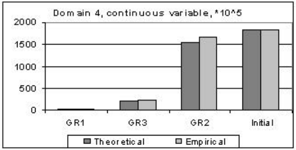

Domain 1, binary variable
80000,0
60000,0
40000,0
20000,0
0,0
GR1
GR3
GR2
Initial
Theoretical
Empirical

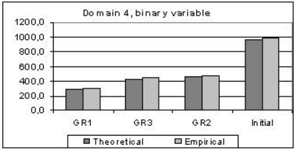

Domain 2, continuous variable, *10^5
10000
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical


Domain 2, binary variable
50000,0
40000,0
30000,0
20000,0
10000,0
0,0
GR1
GR3
GR2
Initial
Theoretical
Empirical


Domain 3, continuous variable, *10^5
8000
6000
4000
2000
0
GR1
GR3
GR2
Initial
Theoretical
Empirical


Domain 3, binary variable
120000,0
80000,0
40000,0
0,0
GR1
GR3
GR2
Initial
Theoretical
Empirical


Domain 4, continuous variable, *10^5
2000
1500
1000
500
0
GR1
GR3
GR2
Initial
Theoretical
Empirical


Domain 4, binary variable
1200,0
1000,0
800,0
600,0
400,0
200,0
0,0
GR1
GR3
GR2
Initial
Theoretical
Empirical

Final conclusion

- Three GR estimators satisfying linear restrictions, generalized to allow biased initial estimators, were proposed.
- The mean square error (MSE) and the bias expressions for the GR estimators were derived and studied.
- The ordering of the GR estimators was established, they all were more accurate than the initial estimator.

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THANK YOU!