

Estimation strategy for small areas, a case study

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Purpose			

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In this research

I am interesting in an overall **strategy**

that deals with small area problems,

involving both planning sample design

and estimation aspects.

Purpose			

Outline

- Notations;
- Sample designs: balanced sample, model-based design;
- Estimators and Models;
- Simulation;
- Conclusions.

Notations			

Finite population: $U = \{1, 2, ..., N\};$

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Finite population: $U = \{1, 2, ..., N\};$

Domain population: $U^{(d)} = \{1, 2, ..., N^{(d)}\}, d = 1, ..., D;$

Study variable: *y*(*t*);

Auxiliary variables: $\mathbf{x}(t) = \{x_1(t), x_2(t), \dots, x_J(t)\} \in \mathbb{R}^J;$

Study parameter - domain population total:

$$TOT^{(d)}(t) = \sum_{k \in U^{(d)}} y_k(t) = \sum_{k \in U} q_k^{(d)} y_k(t), \quad d = 1, \dots, D; \quad t = 1, 2, \dots$$
(1)

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Sample: $S(t) = (S_1(t), S_2(t), \dots, S_N(t));$

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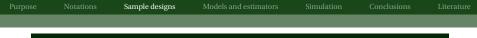
The sample set: $s(t) = \{k : k \in U, S_k(t) \ge 1\};$

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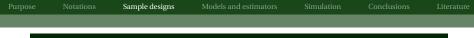
Inclusion probability for unit k: $\pi_k(t) = \mathbf{P}(k \in s(t))$.



Balanced sample

A sample is said to be balanced if, for a vector of auxiliary variable $\mathbf{z}(t) = \{z_1(t), z_2(t), \dots, z_L(t)\} \in \mathbb{R}^L$,

$$\sum_{x \in s(t)} \frac{\mathbf{z}_k(t)}{\pi_k(t)} = \sum_{k \in U} \mathbf{z}_k(t).$$
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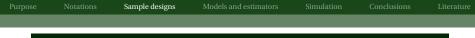
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Cases of the balanced samples

1 Sampling with a fixed sample size if $\mathbf{z}(t) = \pi_k(t)$

$$\sum_{k \in s(t)} \frac{\pi_k(t)}{\pi_k(t)} = \sum_{k \in s(t)} 1 = \sum_{k \in U} \pi_k(t);$$
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2 Stratified sample if
$$\mathbf{z}(t) = \delta_{kh} = \begin{cases} 1, & \text{if } k \in U_h; \\ 0, & \text{otherwise.} \end{cases}$$

	Sample designs		

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- Cube method is general in the sense that the inclusion probabilities are exactly satisfied, that these probabilities may be equal or unequal and that the sample is as balanced as possible;
- It is possible to get SAS/IML version of Cube method done by Chauvet and Tille (2006) and it is also available on the University of Neuchatel Web site. This software program is free, available over the Internet and is easy to use;



 If the sample allocation is small in some strata, balanced sampling will be only very approximate;

		Sample designs	Models and estimators		Conclusions	Literature
Ba	lanced sa	mnle might r	not heln to impro	we estimat	tion	

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- If we can reasonably assume that the balancing variables are no longer correlated to the variables of interest. This can occur when the balancing and the variables used in estimation stage are the same variables measured at different moments.

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Balanced sample might not help to improve estimation						

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Model - based design (Nekrašaitė-Liegė, Radavičius, Rudys, 2011)

The suggested model-based sample design consists of three steps:

- Model construction and estimation of it's coefficients;
- **2** Estimation of the variance of the prediction error;
- **3** Construction of the sample design p(.).

	Sample designs		

Estimators:

Horvitz-Thompson (H-T) estimator

$$\widehat{TOT}_{H-T}^{(d)} = \sum_{k \in s(t) \cap U^{(d)}} \frac{y_k(t)}{\pi_k(t)};$$
(4)

Generalized regression (GREG) estimator

$$\widehat{TOT}_{GREG-\mathcal{M}_{l}}^{(d)}(t) = \sum_{k \in U^{(d)}} \hat{y}_{k}(t) + \sum_{k \in s(t) \cap U^{(d)}} \frac{(y_{k}(t) - \hat{y}_{k}(t))}{\pi_{k}(t)} \quad l = 1, \dots 7.$$
(5)

	Models and estimators		

Fixed effect models

$$\mathcal{M}_{1}:Y_{k}(W) = \beta_{0}(W) + \sum_{j=1}^{J} \beta_{j}(W)X_{j,k}(W) + \varepsilon_{k}(W), \quad k \in U;$$
(6)
$$\mathcal{M}_{2}:Y_{k}(W) = \beta_{0,g(k)}(W) + \sum_{j=1}^{J} \beta_{j}(W)X_{j,k}(W) + \varepsilon_{k}(W), \quad k \in U;$$

$$\mathcal{M}_{3}:Y_{k}(t) = \beta_{0,g(k)} + \sum_{j=1}^{J} \beta_{j,g(k)}X_{j,k}(t) + \varepsilon_{k}(t), \quad k \in U;$$

$$\mathcal{M}_{4}:Y_{k}(t) = \beta_{0,g(k)} + a_{0}^{(d)}t + \mathbf{a}^{\prime(d)}\alpha(t) + \sum_{j=1}^{J} \beta_{j,g(k)}x_{j,k}(t) + \varepsilon_{k}(t), \quad k \in U.$$

	Models and estimators		

Random effect models

$$\mathcal{M}_{5}:Y_{k}(W) = \beta_{0}(W) + r_{0,g(k)}(W) + \sum_{j=1}^{J} \beta_{j}(W)X_{j,k}(W) + \varepsilon_{k}(W), \quad k \in U;$$

$$(7)$$

$$\mathcal{M}_{6}: Y_{k}(t) = \beta_{0}^{(d)} + r_{0,g(k)} + \sum_{j=1}^{J} \beta_{j}^{(d)} X_{j,k}(t) + \varepsilon_{k}(t), \quad k \in U;$$

$$\mathcal{M}_{7}: Y_{k}(t) = \beta_{0}^{(d)} + r_{0,g(k)} + a_{0}^{(d)} t + \mathbf{a}^{\prime(d)} \alpha(t) + \sum_{j=1}^{J} \beta_{j}^{(d)} x_{j,k}(t) + \varepsilon_{k}(t), \quad k \in U.$$



Simulation

Population: Lithuanian survey on short-term statistics on service (N = 750); Time: each quarter from 2005 till 2009;

Study variable y(t): income; Auxiliary variables $\mathbf{x}(t)$: number of employees $(x_1(t))$, VAT $(x_2(t))$, NACE code, region indicators $(x_{3,1}(t), x_{3,2}(t), \dots, x_{3,11}(t))$;

			Simulation	
Simula	ation			

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Sample designs: SRS (n = 300, M = 1000); SSRS (n = 300, M = 1000), strata - size of enterprise; M-B (n = 300, M = 1000);

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Sample designs: SRS (n = 300, M = 1000); SSRS (n = 300, M = 1000), strata - size of enterprise; M-B (n = 300, M = 1000); Balanced variables: $\pi_k(t), x_{1k}(t), x_{2k}(t)$;

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> Sample designs: SRS (n = 300, M = 1000); SSRS (n = 300, M = 1000), strata - size of enterprise; M-B (n = 300, M = 1000); Balanced variables: $\pi_k(t), x_{1k}(t), x_{2k}(t)$; Models: $\mathcal{M}_1 - \mathcal{M}_7$; Estimators: H-T, GREG- \mathcal{M}_l l=1,...7.



Research 1: Accuracy measures

Absolute relative bias:

$$ARB = \frac{\left|\frac{1}{M}\sum_{m=1}^{M}\widehat{TOT}_{GREG-\mathcal{M}_{l}}^{(d)(m)}(t) - TOT^{(d)}\right|}{TOT^{(d)}};$$
(8)

Relative root means square error:

$$RRMSE = \frac{\sqrt{\frac{1}{M}\sum_{m=1}^{M} (\widehat{TOT}_{GREG-\mathcal{M}_{l}}^{(d)(m)}(t) - TOT^{(d)})^{2}}}{TOT^{(d)}}.$$
(9)

 $\widehat{TOT}_{GREG-\mathcal{M}_l}^{(d)(m)}(t) \text{ - replicates of the estimates } \widehat{TOT}_{GREG-\mathcal{M}_l}^{(d)}(t), \\ m = 1, ..., M, \ l = 1, ..., 7.$

		Simulation	

Simulation results1: SRS, population case

	Sample design, balanced variables						
Estimator	SRS, $\pi_k(t)$		SRS, π	SRS, $\pi_k(t)$, $x_{1k}(t)$		$, x_{1k}(t), x_{2k}(t)$	
	MARB, %	MRRMSE,%	MARB, %	MRRMSE,%	MARB, %	MRRMSE,%	
H-T	0.5	14.7	0.5	11.4	0.5	9.8	
GREG- \mathcal{M}_1	0.4	7.0	0.4	7.0	0.4	6.7	
GREG- \mathcal{M}_2	0.4	7.0	0.4	7.0	0.4	6.7	
GREG- \mathcal{M}_3	0.2	6.0	0.3	6.1	0.3	5.9	
$\text{GREG-}\mathcal{M}_4$	0.2	6.2	0.4	6.5	0.3	6.2	
GREG- \mathcal{M}_5	0.3	6.9	0.4	6.8	0.3	6.5	
GREG- \mathcal{M}_6	0.3	6.3	0.4	6.5	0.2	6.2	
GREG- \mathcal{M}_7	0.2	6.2	0.4	6.5	0.3	6.2	

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Simulation results1: SRS, population case

	Sample design, balanced variables							
Estimator	SRS	$\delta, \pi_k(t)$	SRS, π	$k(t), x_{1k}(t)$	SRS, $\pi_k(t)$	SRS, $\pi_k(t), x_{1k}(t), x_{2k}(t)$		
	MARB, %	MRRMSE,%	MARB, %	MRRMSE,%	MARB, %	MRRMSE, %		
H-T	0.5	14.7	0.5	11.4	0.5	9.8		
GREG- \mathcal{M}_1	0.4	7.0	0.4	7.0	0.4	6.7		
GREG- \mathcal{M}_2	0.4	7.0	0.4	7.0	0.4	6.7		
GREG- \mathcal{M}_3	0.2	6.0	0.3	6.1	0.3	5.9		
GREG- \mathcal{M}_4	0.2	6.2	0.4	6.5	0.3	6.2		
GREG- \mathcal{M}_5	0.3	6.9	0.4	6.8	0.3	6.5		
GREG- \mathcal{M}_6	0.3	6.3	0.4	6.5	0.2	6.2		
GREG- \mathcal{M}_7	0.2	6.2	0.4	6.5	0.3	6.2		

		Simulation	

Simulation results 2: GREG- \mathcal{M}_3 estimator

Sample design,	Domain sample size classes					
balanced variables	Sm	all 0 – 9	Mediu	ım 10 – 19	Larg	je 20 –
	MARB, %	MRRMSE,%	MARB, %	MRRMSE,%	MARB, %	MRRMSE, %
SRS, $\pi_k(t)$	1.7	43.1	1.2	14.4	0.6	10.4
SRS, $\pi_k(t)$, $x_{1k}(t)$	1.7	43.4	1.2	14.6	0.4	10.4
SRS, $\pi_k(t), x_{1k}(t), x_{2k}(t)$	1.8	41.6	0.7	14.4	0.4	10.0
SSRS, $\pi_k(t)$	1.8	18.1	0.6	11.6	0.2	5.5
SSRS, $\pi_k(t), x_{1k}(t)$	1.8	18.3	0.5	11.2	0.2	5.5
SSRS, $\pi_k(t), x_{1k}(t), x_{2k}(t)$	1.8	17.9	0.6	11.0	0.4	5.4
M-B, $\pi_k(t), x_{1k}(t), x_{2k}(t)$	1.6	17.0	0.9	10.5	0.3	5.1

		Simulation	

Simulation results3: Model-based sample design

Model for	Domain sample size classes					
sample design	Sma	ll 0 – 9	Mediu	m 10 – 19	Large 20 –	
and estimator	MARB, %	MRRMSE, %	MARB, %	MRRMSE, %	MARB, %	MRRMSE, %
\mathcal{M}_3	1.6	17.0	0.9	10.5	0.3	5.1
\mathcal{M}_4	1.5	16.7	0.9	10.4	0.4	5.1
Mc	1.6	17.1	0.9	10.5	0.4	5.2
\mathcal{M}_6 \mathcal{M}_7	1.6	16.9	0.9	10.5	0.4	5.1

		Simulation	

Simulation results3: Model-based sample design

Model for	Domain sample size classes						
sample design	Sma	ll 0 – 9	Mediu	Medium 10 – 19		e 20 –	
and estimator	MARB, %	MRRMSE, %	MARB, %	MRRMSE, %	MARB, %	MRRMSE, %	
\mathcal{M}_3	1.6	17.0	0.9	10.5	0.3	5.1	
\mathcal{M}_4	1.5	16.7	0.9	10.4	0.4	5.1	
\mathcal{M}_6	1.6	17.1	0.9	10.5	0.4	5.2	
\mathcal{M}_6 \mathcal{M}_7	1.6	16.9	0.9	10.5	0.4	5.1	



In my research the results showed that:

 the choice of model has bigger effect on estimator then the number of balanced variables;



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- the choice of model has bigger effect on estimator then the number of balanced variables;
- the sample design has bigger effect on estimator then the number of balanced variables;
- **3** the sample design has bigger effect on estimator then the choice of model.

			Literature

Literature

- Chauvet, G., Tillé, Y. (2006). A fast algorithm of balanced sampling. *Journal of Computational Statistics* 21, 9 31.
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			Literature

Thank you